Symposium: Young Children's Transition to Mathematical Drawing

The set of papers comprising this symposium report selected aspects of separate research projects. To address the theme of this symposium - *Young children's transition to mathematical drawing* – each researcher has critically reflected on children's drawings to highlight the diversity in the characteristics of those drawings, and the ways in which they represent (or do not represent) mathematical concepts or processes. In doing so we seek to problematize the expectation that most children will 'naturally' develop drawing skills during pre-school and the first few years of school, that are effective for representing and communicating mathematical meaning.

Although 'drawing' is only specified a few times as a necessary form of representation in Australian Curriculum (Foundation to Year 2), the expectation is emphasised more strongly through the student work samples provided as illustrations of performance standards. For example, of the twelve 'Satisfactory' work samples provided for Year 1 in the Australian Curriculum website (ACARA, 2014), seven of the tasks required drawn responses. The drawings include pictorial representations of quantities, operations and problem solutions, as well as more formal diagrams (number line, graph and a map indicating routes, directions and informal distances). In an assessment situation, what is actually being assessed, the child's mathematical understanding or their drawing skills?

Our concern is that a substantial number of children struggle to develop the required drawing skills, and that many teachers are not aware of the need to explicitly support the development of mathematical drawing. The purpose of this symposium is to draw on some existing research to argue the case for further research that can inform early-years classroom pedagogy designed to obviate the potential learning barrier experienced by many children because of their under-developed drawing skills.

ACARA. (2014). Work sample portfolio: Year 1 Satisfactory. Retrieved 2 December 2017 http://docs.acara.edu.au/curriculum/worksamples/Year_1_Mathematics_Portfolio_Satisfactory.pdf

Paper 1: Jennifer Way. *Two birds flew away: The 'jumble' of drawing skills for representing subtraction Pre-school to Year 1.*

Paper 2: Sarah Ferguson, Jill Cheeseman, & Andrea McDonough. *Children's drawings can be windows into mathematics learning.*

Paper 3: Joanne Mulligan. *Interpreting children's drawings as indicators of mathematical structural development.*

Paper 4: Amy MacDonald & Steve Murphy. *Children's representations of clocks at the start of school.*

Chair/Discussant: Janette Bobis

Key words: EARC Early childhood; REPR Representations; PRIM Primary

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Two Birds Flew Away: The 'Jumble' of Drawing Skills for Representing Subtraction Pre-School to Year 1

Jennifer Way
University of Sydney
<jennifer.way@sydney.edu.au>

This paper contributes to the Symposium: *Young Children's Transition to Mathematical Drawing*, by revealing the diversity of drawn responses to a simple subtraction story. The drawings created by 104 children (aged 4.5 to 8 years) showed an expected age-related progression of representational skill. However, the drawings also revealed the difficulties children encountered in representing the dynamic operation of subtraction, and the persistence of diversity in drawing forms across all age levels.

A naturally developing medium of representation for children is drawing. From around the age of four-years children naturally begin to explore the use of drawing, in iconic, symbolic and even emergent mathematical ways (Machón, 2013). However, the natural development of *drawing as personal expression* is interrupted by the intervention of adults around the time of school entry. Children are, rather suddenly, expected to produce drawings that have specific meaning, represent mathematical concepts and processes, communicate their thinking to others, and make use of formal structures and conventions.

Some of the earliest mathematical drawings expected of young children are the depiction of a quantity (group of items), and the combining (addition) and separation (subtraction) of groups. Children's number sense begins well before school entry, with most toddlers and pre-schoolers able to recognize quantities of two to four items, even before they master the process of counting – the visualising skill known as subitising. Similarly, awareness of 'more' and 'less', and the comprehension of informal addition and subtraction of very small quantities, is typical in children of 3 to 5 years. However, the development of children's representational drawing has been less thoroughly researched (Bobis & Way, 2018; MacDonald, 2013).

This paper explores the drawings created by children (aged 4 to 8 years) to represent a subtractive scenario conveyed through a simple story. The purpose is to reveal the variety in such drawings, and to explore similarities and differences across the age range. The set of drawings has been extracted from a larger study, *Emerging Mathematical Drawings*.

The research is framed by a representational theory for learning mathematics (Goldin & Kaput, 1996). From the representational perspective, the critical importance of representations lies in the fact that mathematics essentially consists of *ideas* that are neither directly visible nor tangible, that is, abstract. These representations exist internally (such as mental images, concepts and relationships), and can also manifest as self-created external representations (such as movements and gestures, drawings, models or verbal descriptions). Potentially, we can infer children's internal representations from the external representations they produce (Goldin & Shteingold, 2001). However, children need assistance to connect their representations (both internal and external) to mathematical concepts in more explicit ways – a process often referred to as 'mathematising' (Ginsburg, Lee & Boyd, 2008). The focus of this study is on young children who have little or no experience of explicit coaching in mathematising their drawings (Pre-school), and those who have begun such experiences in formal schooling.

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Method

The site for the 2017 study was a state primary school with an attached Preschool, in the metropolitan area of Sydney. The class teachers for three Preschool, Kindergarten (Foundation), and Year 1 classes (total 9 classes, 104 children), used a script provided by the researcher to ask the children to create a drawing.

Teacher Script:

Listen to this little story. Then I'm going to ask you to draw what happened.

Five birds perched in a row along the top of a fence. Two birds flew away.

Repeat the story, then ask them to '*draw what happened in the story'*.

The decision was made to exclude a final question (e.g. How many birds were left?) to avoid emphasis on just the remaining group, and so encourage attention to the dynamics of the story. Each drawing was labelled with a code indicating the class and child, and their age in years and months. Analysis of the drawings took the form of repeated sorting, attending to similarities and differences in specific features, until groupings emerged. Both pictorial features (e.g. birds, fence) and mathematical features (e.g. groupings, number of 'birds') were observed. Other features such as arrows to depict movement were also noted. Further examination of the groupings led to refinement of the sorting, then clustering to form four broader categories. The categories were named and described, and sub-categories noted. The child's class, age and drawing category code were entered into a spreadsheet to facilitate sorting and sequencing to search for patterns and progressions.

Findings

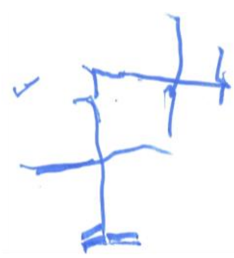
Types of Drawings

Only the four broad categories of drawings are reported here, and have been arranged in sequence from the least coherent to most coherent in terms of mathematical representation of the 'subtractive story'.

Category 1: Scribble. Twelve drawings (12% of 104 drawings) were incoherent, in that there was no apparent representation of the story or depiction of number. This included to two blank pages. Four drawings were literally 'scribble', consisting of seemingly random swirls and lines (see Figure 1). Another seven drawings showed some form or structure, but no recognisable features of the story were discernible (see Figure 2).

(PBB1)

Figure 1. Category 1 - Scribble without form.



(PBB8)

Figure 2. Category 1 - Scribble with some form.

Category 2: Picture. This category contained 21 drawings (20%) that showed the fence and/or birds from the story, but the neither the number of birds nor the number of groups, connected to the quantities in the story. For example, four drawings showed only one bird, seven drawings showed six birds, and one drawing contained 12 birds (Figures 3).

Category 3: Partial Story. These 27 drawings (26%) focused on a particular part of the numerical story by showing one group of birds - either five birds in a single group, or only the two birds that flew away (Figure 4). Some of the drawings included the fence, others did not.


(KG11)

Figure 3. Category 2 – Picture of story element.

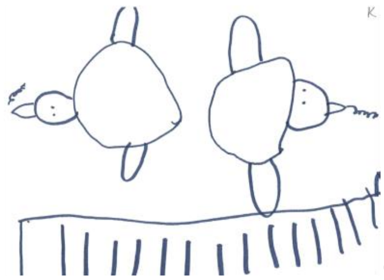

(KG1)

Figure 4. Category 3 – Partial story using number.

Category 4: Partition and solution. All 44 of these drawings (42%) clearly represented the partitioning of a group of five into sub-groups of three and two to reveal the 'solution to the problem'. However, there were several distinct ways of showing the separation. Twenty-eight children drew five birds with two birds clearly positioning above the fence, or crossed out, or identified by upward arrows (See Figure 5). Five children drew just the three birds that were left. Some children captured two events in the story, either with two separate drawings, or with five birds sitting on the fence *and* two flying away (Figure 6).


(KC2)

Figure 5. Category 4 – Partition of 5.


(1D6)

Figure 6. Category 4 – Partition, story events.

Age Groups and Drawing Types

Age groups spanning one year were used, but, as there were only seven children older than 7 years, they were included with the preceding age group (See Table 1). Percentages have been used to facilitate comparisons in both the table and the graph (Figure 7).

Table 1. *Percentage of each Age Group Who Produced each Category of Drawing*

Age	Number of Children	Drawing Type: Percentage (number)			
		Scribble	Picture	Partial Story	Partition
4 to <5 years	27	33 (9)	22 (6)	26 (7)	19 (5)
5 to <6 years	41	2 (1)	29 (12)	32 (13)	36 (15)
6 to 8 years	36	5 (2)	8 (3)	19 (7)	67 (24)
Total	104	12 (12)	20 (21)	26 (27)	42 (44)

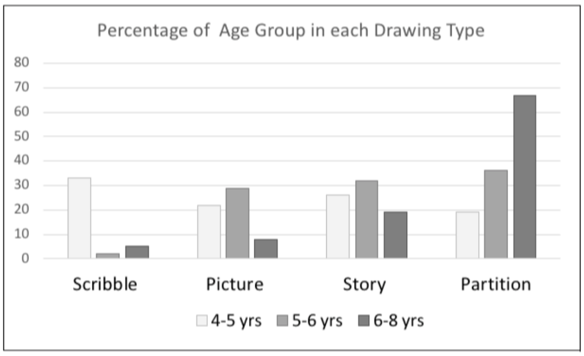


Figure 7. Distribution of age groups across drawing categories

Children from all three age groups produced drawings in all four categories. An increase with age in drawings depicting the partitioning process of subtraction, is clearly apparent. The 4-5 age group is well-spread across all categories, and the 5-6 age group is spread almost equally across the Picture, Story and Partition categories.

Discussion and Conclusion

Both the story and the operation of subtraction are dynamic processes, but a 'one scene' drawing is a static representation. Therefore, the task is quite challenging for young children. The mathematical content of the bird story aligns with the NSW Syllabus (BOS, 2012) expectations for Kindergarten (approx. 5 years). However, the cognitive challenge is increased by having to listen to and process the story information, think of what to draw, and then draw it. It is important to remember that a child may understand both the story events and the mathematics, yet be *unable to draw a representation* of their understanding.

The findings suggest that during the first year of school, many children successfully begin the transition from 'drawing as personal drawing expression' towards mathematical representation of partitioning, yet a substantial number of children are still struggling with the transition midway in their second year of school, when they are expected to use more sophisticated mathematical representations such as empty number lines.

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Children's Drawings can be Windows into Mathematics Learning

Sarah Ferguson
Clairvaux Catholic School
<sferguson@ccsbelmont.com.au>

Jill Cheeseman
Monash University
<Jill.Cheeseman@monash.edu>

Andrea McDonough
Australian Catholic University
<andrea.mcdonough@acu.edu.au>

Using the PPELEM (Pupil Perceptions of Effective Environments in Mathematics) drawing and description tool, 208 children in their first two years of school portrayed themselves learning mathematics well. The responses provided insights for teachers and researchers into children's perceptions of mathematics learning and often revealed a detailed and accurate recall of mathematical events, the locations in which they occurred, the people who were involved, and the mathematics learned.

Children's perspectives of learning are an area of interest for both researchers and teachers who seek insights into children's thinking and feelings about learning experiences (Christensen & James, 2000). However, there are difficulties associated with seeking such insights from young learners.

Drawing holds promise as a research tool for investigating young children's thinking as it can reveal growing conceptual understanding. For example, the Pengelly (1985) "Draw a clock" task demonstrated how children aged 3 to 7 years could represent their understanding of time through drawing. MacDonald and Lowrie (2011) found that children aged 4 to 6 years could represent their growing understanding of length through *drawing telling* as Wright (2007) termed it, which allowed children to "create and share meaning using both verbal and non-verbal modes" (MacDonald & Lowrie, 2011, p. 8). Drawing can also shine light on more affective aspects of children's experiences such as student views on the nature of mathematics (e.g., Solomon & Grimley, 2011) and students' perceptions of changes in education such as the role of the teacher (e.g., Haney, Russeo, & Bebell, 2004). Einarsdottir. Dockett and Perry (2009) used drawing and the child's related narrative to gain insights into how children aged 4 to 6 years viewed starting school. It is apparent that children's drawings and associated narratives can be a window for teachers and researchers into children's understandings and how a child thinks and feels about learning.

Pupil Perceptions of Effective Learning Environments in Mathematics (PPELEM) is a drawing and description instrument developed by McDonough (1992, 2002) to discern student perceptions of effective learning environments in mathematics. McDonough and colleagues have used it as a research tool with students in Years 1 to 6 (e.g., McDonough & Pavlou, 1994) and used an adaptation of PPELEM with teachers (Ferguson, 2011). PPELEM potentially provides insights into preferences and needs of respondents.

In the context of a research project titled *Fostering Inquiry in Mathematics (FIiM)*, PPELEM was used to investigate perceptions of effective mathematics learning situations held by 208 children of 5-7 years. Teachers of Foundation and Year 1 classes in three Victorian schools had been experimenting with adding open-ended activities, problems and investigations to their teaching programs. The children's PPELEM data were intended to complement classroom observation data and testing of children's competencies.

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Method

Teachers collected PPELEM responses by giving the following instruction with pauses between sentences, "I am going to ask you to draw a picture. First, I would like you to close your eyes. Think of times when you have been learning maths and choose one time and place when you are learning maths well. Make a picture of that time in your head. Think about who was there and what was happening." Following the completion of the drawing, children were asked to describe the situation they had portrayed, either through writing or transcription by the teacher.

Analysis and discussion

The PPELEM responses were analysed in relation to a framework of categories. Initially we used categories developed by McDonough (1992) and adjusted them as necessary. After coding a sample of five PPELEM drawings and descriptions, the authors refined and adjusted the categories to suit the responses of young children with the final analysis categorising children's portrayal of location, people and interactions, tools, and mathematical content. We coded a sample of 20% and found an inter-rater reliability agreement of 86.3%.

Several themes emerged from our analysis of 208 drawings and accompanying annotations. We noted that 5-7 year-old children had clear memories of events, and largely pictured themselves participating in mathematical activity and engaging in mathematics with others: classmates, teachers and parents. The clarity of children's memories was a striking feature in the data. For example, one child wrote "I was counting in 10s on the calculator. After 100 I thought it was 200 but it's actually 110". The calculator had allowed her to discover for herself the next number in the sequence and surprise and excitement were evident as she recalled this event. In another example, a child said "I can count to eleven now using my hands. I use ten fingers and one more." Another child described some mathematics from the past and could recall both what he did and who he worked with: "I like measuring because you will get to measure stuff like table, chairs everything and it was not today. It was long, long ago, very long. I was measuring with Joanna and Sui". This supports the findings by Cheeseman (2008) who found that young children could recollect often with clear detail, events from mathematics lessons.

Children's responses often showed them actively participating in mathematics, using manipulatives and undertaking activities. Mathematical manipulatives such as countable objects and geometric materials featured most often (66 times), and 51 drawings included representations such as number lines, number charts, tens frames and cards. Fifty responses gave examples of counting activities (including 27 of skip counting) and objects used for counting were drawn by 34 children. As counting is a focus of the *Australian Curriculum: Mathematics* in Foundation, it receives much attention in the first two years of school. Children have many opportunities to count collections, to structure material to count them efficiently and to count to solve early operations, with this reflected in some children's responses including, "When we counted the days of school I learnt how many days we've been at school". However, the children saw mathematics learning in broader terms than counting and numbers as they drew materials for geometry (10), measurement (8), probability, pattern, time, money, "sharing" and adding (coded as Other manipulatives: 12).

The people the children chose to represent provided some interesting insights. In 62 drawings there was only the child in the picture. Of course, we did ask them to draw a time when *they* “were learning maths well” so it is unsurprising that children drew themselves. This may indicate that children feel they are “doers” of mathematics. However, it was notable that in a further 57 drawings the child drew themselves interacting with other children. Mathematics was seen as something done alone and with others.

Adults had an important role also. Both parents (33) and teachers (31) were portrayed interacting with the child. An interesting finding was how infrequently (4) children drew their mathematics learning happening in a class setting. Learning something with a teacher was often portrayed as learning individually with their teacher. For example, Figure 1:

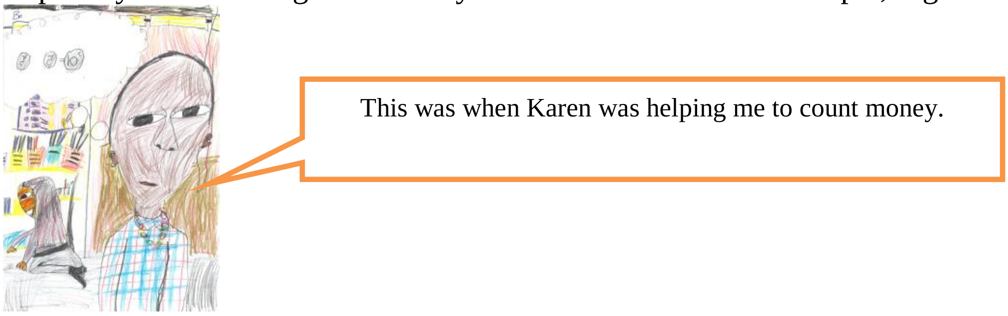

Figure 1. Teacher in the background helping to count money.

Both mothers and fathers were drawn interacting with children and doing mathematics together. Sometimes the whole family was in the picture and it was often clear that the parents were encouraging their child by challenging them to remember facts or to write numbers. In all but two drawings the parents were interacting with the child. In a particularly memorable drawing a five-year-old boy drew himself in the category we called “Location other”. He was in a limousine on the way to his mother’s wedding trying to make her feel happy by counting to her by threes. Most of the drawings in the “Location other” category were in more everyday places such as shops and supermarkets and at the beach (11) but in each drawing the family was involving the child in practical mathematics. For example, Lucy drew a time at the supermarket (Figure 2):

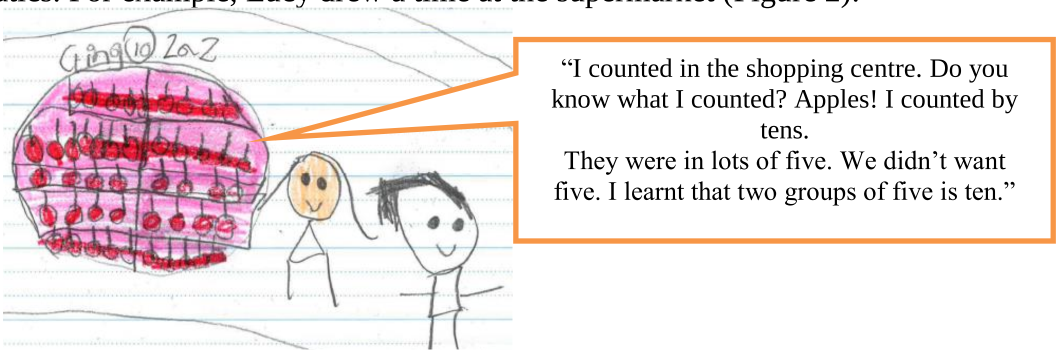

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Figure 2. Lucy counting apples in the supermarket.

Impact of PPELEM – a classroom teacher’s perspective

The theme that stood out for me from the PPELEM drawings and descriptions completed by my class was the prominence of calculators. The children were able to clearly tell me what they were learning when they used the calculator such as “I learnt that 800 plus 800 is 1600!” said with delighted surprise. Using the calculators to skip count and investigate patterns was one of the suggested tasks from the FIiM project in which I was involved. Clearly this task resonated with the children who recalled with photographic clarity participating in the task, could articulate clearly what they had learnt, and communicated their enthusiasm and engagement with using calculators.

My reflections about the children's PPELEM responses featuring calculators led to the following conclusions: the children had certainly learnt and explored important mathematical ideas as a result of using the calculators; the calculators allowed them to investigate numbers which we had not previously talked about in the classroom such as larger numbers and negative numbers; and their engagement was high when using calculators. Further reflection on the children's engagement led me to believe that the individual choices the calculators allowed the children to make gave them a sense of agency and control which they enjoyed. As a result of the PPELEM responses I used calculators far more than in previous years when teaching young children and also made them available for children to use during any 'free time' they had.

Conclusion

Drawing tools like PPELEM have the power to provide insights and windows into children's thinking about mathematics learning. The results from young children's drawings show that young children are capable of recalling in detail past mathematical events and communicating what they learnt from them. Children largely drew themselves actively participating in mathematics and engaging in mathematics with others: children, teachers and parents. This in turn influenced their teachers as they gained insights into the contexts, tasks and tools that had an impact on their students' learning of mathematics.

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Interpreting Children's Drawings as Indicators of Mathematical Structural Development

Joanne Mulligan
Macquarie University Sydney
<joanne.mulligan@mq.edu.au>

This paper contributes to the Symposium: *Young Children's Transition to Mathematical Drawing*, by providing an analysis of children's drawings of number patterns comparing spontaneous and task-based situations. Data drawn from the *Reconceptualising Early Mathematics Learning Project,* involving 153 Kindergarten children (aged 4.5 to 6 years) were analysed for five increasing levels of structural features. Children's spontaneous drawings of patterns and their self-constructed representation of a number sequence elicited at a task-based interview, achieved 0.82 consistency. The analysis exemplifies that children's self-initiated drawing, and the process of creating these offers reliable and authentic evidence of their developing conceptual structures.

The analyses of children's drawings of their mathematical ideas and solutions to tasks have played an important role in much of the research in early childhood education over the past decades (Worthington & Carruthers, 2005). Children's drawings have featured for example, in the analysis of early inscriptions of number (Hughes, 1986), in studies on story problems (Carpenter, Moser & Bebout, 1988) and the counting sequence (Thomas, Mulligan & Goldin, 2002), and more recently in studies of early algebra, and pattern and structure respectively (Brizuela, 2004; Mulligan, English, Mitchelmore, & Crevensten, 2013). Curriculum developers and professional development programs have also promoted broadly the development of representational thinking through children's drawings and justifications often portrayed as 'work samples'.

Recent shifts in theoretical approaches based on 'embodied action' have re-directed attention to the role of drawings as more than artifacts that are used to assess what children have learned, "*re*presentations that reveal their cognitive schema— what they 'know' about geometry, such as their cognitive capabilities, spatial awareness, and conceptual understanding" (Thom, 2018). Thom and McGarvey (2015) conceived children's mathematical drawings as both acts and artifacts where the act of drawing serves as a means of developing awareness of concepts and relationships rather than being a product of that awareness. Although new research is directing attention to the analysis of the embodied process of drawing, there remain few studies that provide analyses of both the process and the artefact or product, along with the child's explanation and sense making of the process. Ideally, intensive and systematic use of digital recordings would be required to capture longitudinal evidence of developing conceptual structures.

In this paper, I raise the question of how to effectively elicit and interpret children's drawings as authentic indicators of their mathematical development. The distinction is in whether the child initiates and creates the mathematical features depicted in the drawing or whether the drawing is a reaction to, or a replication of an imposed mathematical model, tool or graphical representation. The purpose of analysing and describing structural features of different types of drawings for the same individual is to provide a more coherent and reliable basis for scaffolding children's mathematics learning.

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Background to the study

Several studies of young children's development of pattern and structural thinking in mathematics have been based on exploratory work analysing children's images (self-initiated drawings) and how these reveal critical developmental features of mathematical conceptual development. Thomas, Mulligan and Goldin (2002) analysed drawings (and explanations) of the counting sequence 1-100 from a sample of 172 children from Grades K to 6, and 92 highly able children. Children's understanding of the baseten system was reflected in a wide variety of iconic, pictorial and notational recordings showing how representational systems for numbers may change through a period of structural development to become eventually powerful, autonomous systems. The analysis highlighted the importance of identifying whether the child's drawing reflected a static or dynamic view of numerical sequences.

A series of studies followed (see Mulligan, 2010), informing the development of the Pattern and Structure project, aimed at developing and validating an interview-based assessment of structural development and evaluation of a pedagogical program to promote early awareness of patterns and emergent generalisation. This paper reports one aspect of the analysis of children's representations drawn from the *Reconceptualising Early Mathematics Learning study*, (see Mulligan et al., 2013).

Method

An intervention program focused on developing mathematical patterns and structures across a wide range of concepts was trialled with experimental groups over the entire first year of schooling. Children's responses to interview-based assessment tasks (the Pattern and Structure Assessment [PASA]) and structured tasks, included drawing using paper and pencil. Opportunities for children's spontaneous constructions and drawings of patterns were also integrated into the program.

A representative sample of the drawings and accompanying exemplars of digital recordings of the drawing process from each student for each group of learning experiences were collected and analysed for qualitative differences by the research team. For illustration, the paper provides an overview of a comparative analysis of drawings from 153 children (aged 4.6 to 6 years) of the numerical sequence "9, 10 and 11" along with one example of their spontaneous drawing of a mathematical pattern. The assessment interview task aimed to capture children's self-constructed representations of number for features of pattern and structure such as equal groups or array structure. Another pre-interview task asked the children to draw a mathematical pattern— "anything that shows me clearly what a pattern is". Data was collected at one interview point prior to commencement of the intervention program by trained researchers. The children were required to using pencils and A3-sized paper for recording. Thus, the exemplars are limited to a 'shapshot' of the child's conceptual structure of number and pattern.

Analysis and Discussion of Findings

Recordings of the numerical sequence task were analysed for features of pattern and structure consistent with previous coding for one of five levels of structural development: pre-structural, emergent, partial, structural and advanced structural. Table 1 shows the percentage of drawings categorised by level and a description of the drawings' features.

Table 1.
Drawings of the numerical sequence 9, 10 and 11 by structural level (n=153)

Structural Level	% Drawings	Description
1. Pre-structural	13	Numerals 9, 10 and/or 11 drawn randomly without any representation of order or quantity
2. Emergent	26	Objects, marks or icons randomly drawn to represent the quantities 9, 10 and 11 often accompanied by symbols correctly represented.
3.Partial structural	35	Partially formed groups, rows or arrays drawn with dots, marks, pictures or icons in order, accompanied by correct numerals; or incomplete representation of the sequence
4. Structural	19	Groups of objects, dots, marks or icons correctly forming arrays such a 3 x 3 and systematically ordered with correct use of numerals
5. Advanced	7	All structural features shown with evidence of extending and decomposing the pattern of representations (such as "3 x 3 and two more is 11") and/or creation of application of the numerical pattern such as 19, 20, 21.

The majority of drawings fell into emergent and partial structural levels consistent with other assessment data on the same group of students.

Table 2.
Percentage of spontaneous drawings of patterns by structural level (n=153)

Structural Level	% Drawings	Description
1. Pre-structural	19	Icons, marks or other idiosyncratic features place randomly without evidence of pattern as repetition
2. Emergent	23	Objects, marks or icons randomly drawn to represent a simple repetition but without any consistent spatial structure
3.Partial structural	32	Objects, marks or icons drawn consistently in order to represent simple repetitions usually with an incomplete unit of repeat
4. Structural	20	Groups of objects, dots, marks or icons correctly forming units of repeat to represent simple or complex repetitions including border patterns
5. Advanced	6	Structural features shown with evidence of extending and symbolising the pattern, and expressed as a generalised 'rule'

Table 2 provides a similar analysis of children's spontaneous drawing of a pattern. There were more drawings categorised at the pre-structural level (19%) compared with the numerical sequences at the same level (13%). However these findings were consistent for the emergent and partial structural level for both tasks. Analysis of individual patterns of response found a 0.82 level of consistency.

The levels of structural development depicted by the drawings provided partial evidence of the child's developing mathematical structures. In combination with analysis of assessment data and a range of other drawn and verbal responses over time a more coherent profile of development can be built. In the present study an overall level of Awareness of Mathematical Pattern and Structure (AMPS) was measured and described. Supporting evidence from the drawings was critical in the formation of a reliable measure of AMPS. Although the analysis did provide explicit indicators of developing structures it could not be assumed that the origins of these representations were created entirely by the child or what influences impacted on the child's imagery. The question remains whether children re-configure or transform images from their interaction with the real world to represent what they see and mean. Further, depictions in the drawings may have reflected a response that their classroom teacher encouraged or expected, even though the children were at the early stage of formal schooling.

Limitations and Implications

The exemplars analysed in this report present artefacts or products as drawings produced at one point in time. These indicated the presence or absence of important developing structural features such as equal grouping, partitioning, array structure and unit of repeat. Although videos of the patterning process were collected in this study it was not feasible to record and analyse the drawing process for each child. What we did capture were examples of the process of drawing conceptual structures for a representative sample for each conceptual topic over the course of the program. Despite the valuable insights that can be gained from explicit interpretation of drawings, particularly the authentic exemplars self-initiated by the child, drawings in themselves do not provide a coherent picture of children's developing mathematics. Goldin cautions that the researcher must consider that they can only ever make inferences about the child's external representations (drawings) as accurate representations of their internal structures.

In connection and aligned with Way (this symposium) when children begin formal schooling there seems to be a transition from spontaneous self-initiated drawing, and using a broader and unrestricted range of media, to teacher-directed more formal drawings of mathematical ideas or situations, often limited by the size and shape of the media and tools for expression. This paper raises the issue that there remains a stark difference between children's drawings that are 'pedagogically imposed' and those that are spontaneously-elicited images of mathematical ideas. Raising this awareness with both researchers and professionals is critical for further reliable use of drawings as evidence of mathematical development.

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Children's Representations of Clocks at the Start of School

Amy MacDonald
Charles Sturt University
<amacdonald@csu.edu.au>

Steve Murphy
Charles Sturt University
<smurphy@csu.edu.au>

This paper contributes to the Symposium: *Young Children's Transition to Mathematical Drawing*, by examining the extent to which children are able to draw the structural features of a clock at the start of school. The drawings were produced by 132 Kindergarten children in their first six weeks of primary school. The drawings showed that the majority of the children started school with the ability to represent the structural features of a clock (numbers, hands, partitioning).

Background

Time is often seen as a difficult topic by teachers and children throughout primary school (Burny, Valcke, Desoete, & Van Luit, 2013). There is also relatively little research around young children's understandings of clocks. A seminal study was that of Pengelly (1985), who asked children aged 3 to 7 years to create a clock face using a range of materials. Pengelly suggested that children's understanding of the clock face progresses through five developmental stages: 1. Early impressions of a clock; 2. Awareness of the numerals on a clock; 3. Awareness of the importance of the twelve numerals; 4. Partitioning of the twelve numerals becomes significant; and 5. Recognition of minute markers. More recently, Smith and MacDonald (2009) examined the clock drawings of 4 to 6 year olds and noted, in particular, a fixation on the role and movement of the hands of a clock - a finding that challenged Pengelly's developmental sequence, which did not include a focus on hands.

Despite children's early understanding of clock faces, the *Australian Curriculum – Mathematics* (ACARA, 2017) only expects children to be reading clock faces at the conclusion of Year 1, when aged 6 to 7 years. There is no mention of clocks in the curriculum for the Foundation year, just a requirement to sequence familiar events in time. This study examines the extent to which children are able to use drawings to represent the structural features of a clock at the start of school. Specifically, this study considers three key structural features – numbers, hands, and partitioning – and the sophistication with which children are able to represent these features in their mathematical drawings.

Method

This study was part of a wider project undertaken with 132 children who had just commenced Kindergarten (Foundation) at two primary schools in regional NSW. The data were collected within six weeks of the children starting school. The children were simply asked to "draw a clock"; no further instructions were given. Once the drawing was completed, children were invited to explain what they had drawn and the drawings were annotated with this narrative. Only two children chose to draw digital clocks, with the rest of the sample drawing analogue clocks. For this study, analysis is based on features of analogue clocks as represented in the drawings only – independent of the accompanying narrative. The coding was based on three structural features of an analogue clock: numbers, hands, and partitioning. The drawings were coded according to the degree of sophistication of these three features evident within the drawing, as shown in Table 1.

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Table 1

Coding of the structural features of an analogue clock

Numbers	Hands	Partitioning
1. No number representation	1. No indications of hand(s)	1. No partitioning
2. Some number representation	2. Indication of hand(s)	2. Developing partitioning
3. Numbers in sequence	3. Two equal length hands	3. Partitioning
4. Numbers 1-12 in sequence	4. Two (or three) differentiated hands	

Results

The analysis revealed that the majority of children represented one or more structural features of a clock. Only 14 children (11%) were classified as not representing any of the three features. Of these 14, one drew a digital clock (but with no numbers represented), and one chose to draw a cuckoo clock. The remaining 12 drew a vaguely circular form, but with no clearly discernible structural features of a clock. 22 children (17%) represented, to some degree, one of the features. 46 children (35%) represented two features, while the other 50 children (38%) represented at least some indication of all three features.

Representation of numbers

Only 17 children (13%) did not make some representation of numbers in their drawings. These children did, however, represent at least one other feature, such as the hands (Figure 1). 42 children (32%) represented numbers in some form; usually through the use of dots or dashes (Figure 2), or through identifiable numerals. The majority (57 children; 43%) not only represented numerals, but also represented these in a sequence (Figure 3). Often these sequences extended beyond the number 12. Finally, at the highest level of sophistication, 16 children (12%) clearly represented the numerals 1-12 on their clock face (Figure 4).

Figure 1. No representation of numbers.

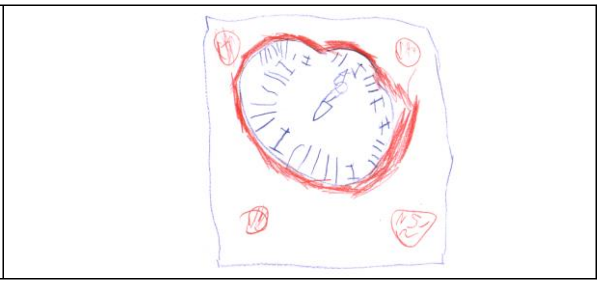

Figure 2. Representation of numbers.

Figure 3. Representation of a number sequence.

Figure 4. Representation of numbers 1-12.

Representation of hands

The majority of the children gave some indication of clock hands in their drawing; although, 45 children (34%) did not represent hands in any way (Figure 5). Of those who did represent hands, most (37 children; 28%) used marks to indicate the position of at least one hand; sometimes more than three hands were evident (Figure 6). 20 children (15%) represented two hands of the same length, with no differentiation between hour hand and minute hand (Figure 7). Nearly a quarter (30 children; 23%) of the drawings clearly represented dimorphic hands (Figure 8), with some children also including a seconds hand.

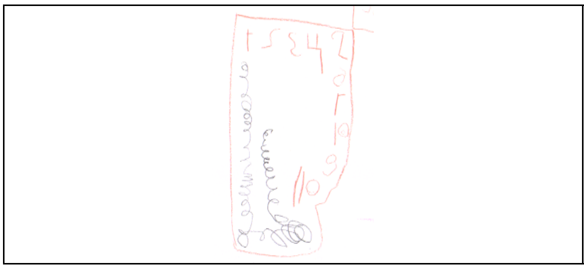

Figure 5. No representation of hands.

Figure 6. Indication of hands.

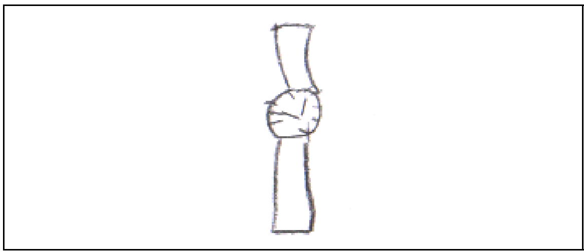

Figure 7. Two equal-length hands.

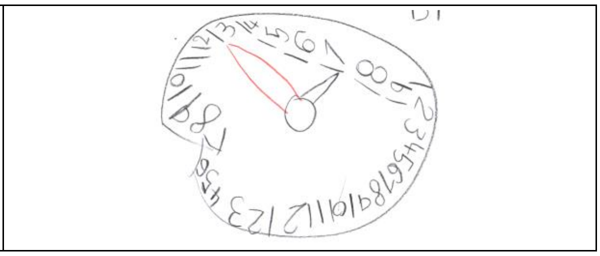

Figure 8. Two (or three) differentiated hands.

Representation of partitioning

Drawings were classified as having no partitioning evident if the numerals/marks were placed haphazardly, or around an arc of the clock face (Figure 9). This was characteristic of most of the drawings, with 70 children (53%) coded as not representing partitioning. Inversely, partitioning *was* evident in nearly half of the sample. 48 children (36%) showed a developing sense of partitioning. Drawings were coded as "developing" when there was an attempt to evenly place numerals/marks around the clock face (Figure 10). Some responses also showed a need to "fill the face", i.e. have numerals/marks all the way around the clock face. In instances where the children stopped at 12 (or another number, i.e. 19), attempts were made to "fill the gap" with scribbling, colouring, or the placement of the hands in the space left over. Finally, there were 14 children (11%) who clearly represented partitioning of 12 numerals/marks around the clock face (Figure 11).

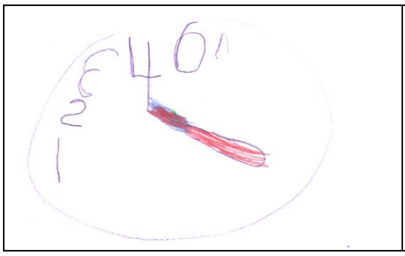

Figure 9. No partitioning.

Figure 10. Developing partitioning.

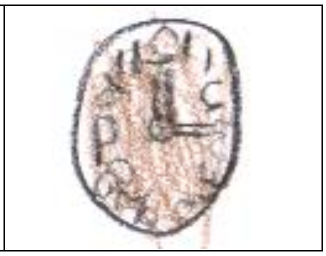

Figure 11. Partitioning.

Discussion and Conclusion

The results showed that the majority of children start school with some ability to represent the structural features of a clock (numbers, hands, partitioning), with 117 children (89%) representing at least one structural feature in their drawing.

It was logical that children who did not represent numbers in any way were also classified as not representing partitioning. Only 19 children (14%) represented the numbers only, with no indication of hands or partitioning. 87 children (66%) represented hands, consistent with Smith & MacDonald's (2009) finding that many children recognise the hands as a feature of clocks. Interestingly, there was a relationship evident between number sequencing and developing partitioning, with 31 children demonstrating these two categories. Typically, these children continued or repeated a number sequence to continue their partitioning of numerals right around the clock face. Encouragingly, five children represented all three features at the highest level of sophistication.

Our analysis suggests that children's ability to represent clock structure does not progress linearly, as posited by Pengelly (1985). While some form of number representation is necessary to demonstrate partitioning, some children showed partitioning without any number sequence. Some children demonstrated clock hand differentiation without any number representation, while others represented the numbers 1 to 12 without drawing hands or demonstrating any partitioning. The drawings suggest that different children attend to different features of clocks, and thus have different developmental journeys.

This study demonstrates that many children arrive at school with a sophisticated understanding of clock features; yet, the *Australian Curriculum – Mathematics* (ACARA, 2017) makes no explicit mention of clocks for children just beginning school. This is consistent with international research that indicates a mismatch between the intended mathematics curriculum for the first year of school and children's mathematical ability when starting school (Perry, MacDonald, & Gervasoni, 2015). This presents a risk of these students becoming bored or disengaged with mathematics upon school entry.

The children's drawings present a number of opportunities for mathematical development in the first year of school. For example, clock drawing is means of supporting children's writing of number sequences in a meaningful context. Children can also be supported to develop skills in partitioning and spatial representation. These skills also lend themselves to the representation of other mathematical concepts, such as division and fractions. The "draw a clock" task could easily be utilised in Kindergarten classrooms as a means of ascertaining a foundation for further mathematics learning in the first year of school.

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